Path Planning for Redundant Mobile Manipulator

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Abstract— **This paper presents a path planning methodology for a mobile manipulator system including a car-like mobile robot and a redundant manipulator. The mission is to follow an imposed trajectory by the end-effector. The mobility is insured by the platform in order to make a manipulator able to act outside its workspace.**

Motion planning is computed for the nonholonomic mobile system so that it allows the manipulator to follow the requested trajectory. When the platform is positioned in an appropriate location, an iterative method has been used to configure the manipulator to accomplish the required task. Actual runs and results of our approach are given at the end of the paper.

Index Terms— **Mobile manipulator, motion planning, nonholonomic platform, redundant arm.**

I. Introduction

A mobile manipulator system consists of coupling manipulation represented by an articulated system, and locomotion with using a mobile robot [2] ; this system is presented as a vehicle carrying a manipulator, this alliance has an advantage because it permits to enlarge the workspace of the robotic arm, but it can be an inconvenience in path planning; it is a persistent problem when the manipulator end-effector trajectory is imposed, in this case, the mobile robot can have an infinity of configurations (positions and orientations), and often, the inverse kinematic model of the manipulator is not known explicitly.

Hu and Guo[1] proposed to model a system including a three degrees of freedom (d.o.f) arm with revolute joints, and a nonholonomic platform. They started by the presentation of the forward kinematic model, from which the differential kinematic equations have been deduced. The calculation of the inverse kinematic model has been accomplished analytically thanks to the forward model with imposing orientation of the platform. Finally, a dynamic model of a system has been presented. The same approach that is decoupling the combined system has been adopted by Papadopoulos and Poulakakis [2], two kinds of mobile manipulators have been considered, so, they studied the most commonly available mobile platforms which use either a differential drive or a car-like drive. Each of them have its own differential kinematic model, even if they are equipped with the same two-link manipulator. The authors presented a planning methodology for such systems allowing them to follow simultaneously desired end-effector trajectory and a platform path without violating nonholonomic constraints. The executing task is a robotic crack-sealing, while the end-effector must follow some crack on the pavement, and a mobile platform is required to follow a given path, while respecting a condition that the distance between the end-effector and a reference point on the platform is within the reach of the manipulator.

Xu and al [3] presented a particular approach to compensate the gaps of a manipulator having a deficit in d.o.f, for a task getting used in a three-dimensional space. The considered arm has five d.o.f, and the platform is differential drive (PioneerII). The problematic was to consider redundancy after having included the platform, since this one adds three degrees of freedom. The proposed solution was to find an adequate distribution of d.o.f to the combined system, in order to accomplish the recommended actions correctly (opening a door).

Seraji [4] considered a manipulator having two degrees of freedom carried by nonholonomic platform. He studied the two sub-systems, and used the augmented jacobian matrix in order to calculate the inverse kinematic model. He incorporated the nonholonomic constraints with the operational task. The redundancy introduced by the mobility of the platform has been exploited to execute a set of additional tasks during the motion of the end-effector.

The aim of the following work is to calculate for a mobile manipulator the set of configurations needed to follow a predefined operational trajectory. The considered system consists in a nonholonomic mobile platform carrying a redundant manipulator in order to effect a desired task. Our approach consists in planning the motion of the mobile platform with considering the nonholonomic constraints; the main aim of this procedure is to let the manipulator be capable to follow the imposed operational trajectory with placing always the platform consequently. Once this phase accomplished, we use an iterative method in order to configure the carried articulated system. This paper is structured as follow:

Section II: we give a description of the studied system and its characteristics, then we present the direct kinematic model for any mobile manipulator, those two parts are useful for the next sections.

Section III: this section includes the algorithm of our path planning approach, it concerns in calculating the trajectory to be followed by the mobile system according to nonholonomic constraints.

Section IV: the iterative method for the inverse kinematic model of the arm presented in section II, is given with using the direct kinematic model of our mobile manipulator.

Section V: we present in this section the simulation tests of our approach.

II. Direct Kinematic Model

A. The studied system

The mobile manipulator consists of a car-like mobile platform, with two steering front wheels and two rear wheels which are parallel to its main axis, carrying a seven d.o.f redundant arm (Mitsubishi PA10 7CE). In our case, we have considered only the sub-chain $Ang=[q_1\ q_2\ q_3\ q_4]^T$ responsible of positioning the end-effector [5] ; this manipulator is placed in the center of the front wheels axis. Below, we give the scheme of the system in Fig.1.

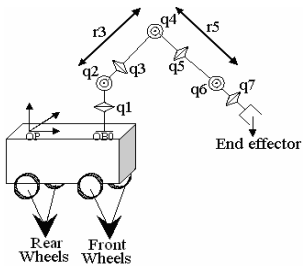

Fig. 1. The Mobile manipulator

B. Presentation of the reference frames

The direct kinematic model of the mobile manipulator needs to focus on three main reference frames.

Absolute Reference Frame: $R_A=(O_A,\vec{x}_A,\vec{y}_A,\vec{z}_A)$ indicates the notation in which must be expressed the position coordinates $[A_1\ A_2\ A_3]^T$ of the end-effector, according respectively to $\vec{x}_A$, $\vec{y}_A$ and $\vec{z}_A$ axis, as well as, its orientations $[A_4\ A_5\ A_6]^T$ with respect to the same three reference axis. R_A is chosen orthonormal, direct, and $\vec{z}_A$ is perpendicular to the surface on which moves the wheeled platform [8].

Platform Reference Frame: The mobile robot must have a reference frame $R_P=(O_P,\vec{x}_P,\vec{y}_P,\vec{z}_P)$, which is represented in R_A by the position $[X_P\ Y_P]^T$ of its origin O_P, and by its orientation α

Manipulator Reference Frame: The reference frame $R_{B0}=(O_{B0},\vec{x}_{B0},\vec{y}_{B0},\vec{z}_{B0})$ is assigned to the manipulation basis where must be calculated the cartesian coordinates $[X_E\ Y_E\ Z_E]^T$ of the end-effector, and its orientations $[\psi\ \theta\ \phi]^T$ given with Euler angles[6]. This model is calculated using the Denavit–Hartenberg notation to obtain the transformation matrices [9]. The cartesian coordinates of the origin O_{B0} are $[a'\ b']^T$ represented in the platform reference frame R_P according respectively to the $\vec{x}_P$ and $\vec{y}_P$ axis. Those principal frames are presented in Fig.2.

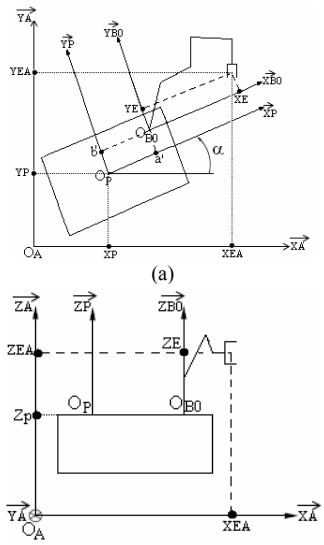

Fig. 2. Presentation of reference frames:
(a) Projection in $(O_A,\vec{x}_A,\vec{y}_A)$, (b) Projection in $(O_A,\vec{x}_A,\vec{z}_A)$

C. Expression of the direct kinematic model

The direct kinematic model is a mathematical representation of the position coordinates A_1, A_2, A_3, and the orientation coordinates A_4, A_5 and A_6 in the absolute reference frame R_A, the equations (1-6) representing this model are as follows [6][7] :

$$A_1 = X_P + (a'+X_E)\cos\alpha - (b'+Y_E)\sin\alpha \quad (1)$$

$$A_2 = Y_P + (a'+X_E)\sin\alpha + (b'+Y_E)\cos\alpha \quad (2)$$

$$A_3 = Z_E + Z_P \quad (3)$$

$$A_4 = \psi + \alpha \quad (4)$$

$$A_5 = \theta \quad (5)$$

$$A_6 = \phi \quad (6)$$

In equations (1-3), $A_1=X_{EA}$, $A_2=Y_{EA}$ and $A_3=Z_{EA}$.
We can deduce from those equations that, for a mobile manipulator, the position coordinates values A_1, A_2 and A_3 are closely related to the platform configuration $[X_P\ Y_P\ \alpha]^T$, this last adds three d.o.f to the manipulation system and contributes to create redundancy. Including the car don't change the orientation values A_5 and A_6, but its rotation α contributes to change A_4.

III. Motion Planning Of The Mobile Platform

A. Operational trajectory

We define it as being a trajectory that consists of a succession of some points close to one another; the aim of this representation is to allow the end-effector to follow a smoothed imposed trajectory, so that it avoids abrupt changes; those points will be called "*step points*", every step point C has its own cartesian coordinates $C=[X_c\ Y_c\ Z_c]^T$ in R_A. A

carried manipulator must reach all step points representing the operational trajectory.

B. Platform generating trajectory

In our approach, we consider the system as partially decoupled; we have built a predefined path for the platform in order to allow the manipulator to reach all the step points.

Our strategy consists in generating a trajectory for the vehicle with taking into account the properties of each part of the mobile manipulator system, because the car moves only in the plan $(O_A, \vec{x}_A, \vec{y}_A)$, and the manipulator is able to reach a desired height Z_c with its end-effector[3], therefore, the mobile system permits to enlarge the workspace of the manipulator and to bring it back to a configuration $Ang=[q_1\ q_2\ q_3\ q_4]^T$ for reaching the required height, and hence the desired step point.

Given C a step point to reach, it generates a virtual circle named "*field*", R_{max} being its radius; if the car is within the field, the manipulator can reach the desired step point, otherwise, it is necessary to move the platform; see Fig.3 for more clarity.

R_{max} has an opposite relationship with Z_c because when this last one increase, R_{max} decrease, and vice-versa.

In our approach, we identify a maximal distance R_{max} between the origin O_{B0} and C_{XY} (projection of the step point C in the frame $(O_A, \vec{x}_A, \vec{y}_A)$ assigned in Fig.3); it is calculated with considering the maximal length MAX of the arm to reach Z_c. This result can be obtained with placing the platform as far as possible, in this operation, the orientation α of the car is not influent. The calculation of R_{max} is shown on equations (7-9):

$$Z = Z_c - Z_P \tag{7}$$

$$\theta = arccos(Z / MAX) \tag{8}$$

$$R_{max} = MAX\ sin\,\theta \tag{9}$$

The imposed step point can't be reached by the manipulator except if the platform brings back O_{B0} in the field, in this case, the distance D_{BE} between O_{B0} and C_{XY} must be less than R_{max} ($D_{BE} \leq R_{max}$) (Fig.3).

We must make some assumptions for the platform path planning:

-The task must be accomplished in the 3D space with considering only the cartesian coordinates $[X_{EA}\ Y_{EA}\ Z_{EA}]^T$.

-For the carried arm (Mitsubishi PA10 7C), we consider only the four first joint angles $[q_1\ q_2\ q_3\ q_4]^T$ responsible of the positioning of the end-effector[5], then one degree of kinematic redundancy appears.

-A navigation surface of the mobile platform assumed to be plane.

-The motion of the mobile manipulator is free of collision.

-The locomotion system must be solicited because the manipulator can't reach all the required step points.

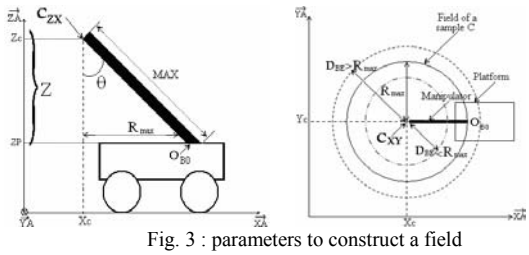

Fig. 3 : parameters to construct a field

C. Trajectory planning of the mobile system

The following algorithm represents the trajectory planning procedure with considering the fields generated by some selected step points:

1-Input an imposed operational trajectory T having N step points, $Dist$, k, Nbr, i.

While $Nbr \leq N$

2-Calculate R_k and R_{Nbr}, field radiuses of step points C_k and C_{Nbr}.

3-Create the segment PR on C_{Nbr} perpendicular to S_i (segment between C_k and C_{Nbr}), and deduce the interaction point P_{Nbr} between the field of C_{Nbr} and PR.

4-Connect P_{Nbr} and P_k and generate τ_t.

5-$k=k+Dist$, $Nbr=Nbr+Dist$, $i=i+1$;

$Dist$ is the distance to separate every couple of step points; k, Nbr, and i are respectively initialised to 1, $Dist$ and 1.

The resulting planned trajectory is a succession of connected segments τ_i representing the path that the mobile system must follow. For more details, see Fig.4.

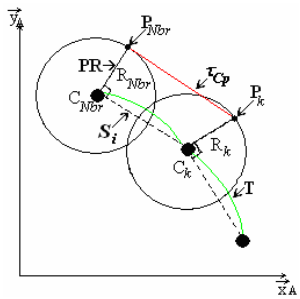

Fig.4. Principal steps in path planning methodology

D. Motion planning

After having estimated a path that the platform must follow with referring to the operational trajectory, we generate the motion of the mobile system with taking into account the nonholonomic constraints. A locomotion system will try to follow a previously generated trajectory.

Latombe and Barraquand [16] proposed a motion planning approach, which considers that a trajectory exists between two platform configurations belonging to the same free space if the steering front wheels angle takes at least two states, for a non-null velocity v.

The equations (10-12) describe the motion planning of a car-like mobile robot presented in Fig.5.

$$\alpha(t) = \alpha(0) + t\,(v / L) \quad (10)$$

$$x(t) = x(0) + (L\ /\ tg\varphi)(sin(\alpha(t)) - sin(\alpha(0))) \quad (11)$$

$$y(t) = y(0) - (L / tg\varphi)(cos(\alpha(t)) - cos(\alpha(0))) \quad (12)$$

L is the distance between the front and the rear axis, $\alpha(0)$ is the initial rotation angle of the mobile robot, $x(0)$ and $y(0)$ represent the initial cartesian coordinates of the reference point O_P. The main aim of this motion generation is to find the time-based coordinates $x(t)$ and $y(t)$ of O_P, and the rotation angle $\alpha(t)$ of the mobile robot, t, considering the car velocity v and the steering front wheels angle φ.

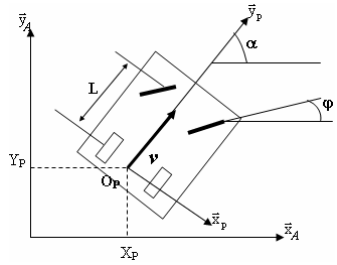

Fig.5 : Representation of a car-like mobile platform

IV. Inverse Kinematic Model Of The Manipulator

Once having the motion planning phase of the platform, we must find the configuration vector *Ang* which is redundant compared to an imposed task, in this way, an iterative method is used to calculate the inverse kinematic model for the articulated system; it is based on the inversion of the jacobian matrix [10].

This method needs to initialise some variables as the initial configuration Ang^0, the vector ε representing the admitted errors when the generalised coordinates in *Ang* are calculated, and a number of iterations k_{max}.

The main steps to calculate the generalised coordinates represented by the vector *Ang* are illustrated in a flow chart of the generalized inverse method (Fig.6). OT^0 represents a vector of cartesian coordinates in R_{B0}, using the configuration Ang^0 and OT^{k-1} is a vector of cartesian coordinates using the configuration Ang^{k-1}

Since that the jacobian matrix is not square, its inverse is calculated using a pseudo inverse of Moore-Ponrose[13].

In the following flow chart, the symbol (1) indicates that J^{-1} is the inverse jacobian matrix calculated from the forward jacobian matrix [11][12], for a configuration Ang^{k-1}.

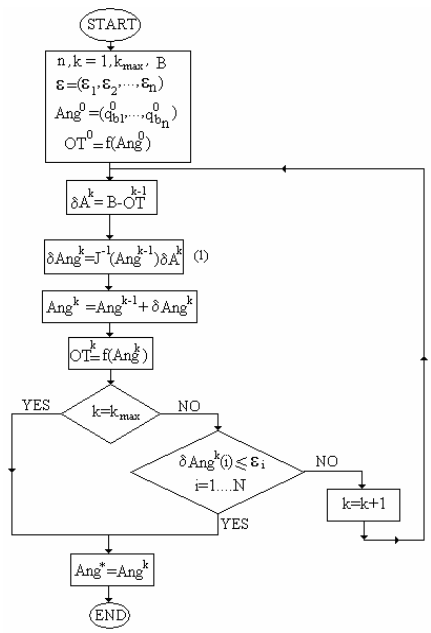

Fig.6 : Flow chart of a generalized inverse scheme

The coordinates step points C (expressed in R_A) are used to calculate the inverse kinematic model, we have to represent them in R_{B0}, this is done with using the equations of the forward kinematic model (1-3) to obtain equations (13-15). The resulting vector is $B=[X_{cb}\ Y_{cb}\ Z_{cb}]^T$.

$$X_{cb} = -a' + (Y_c - Y_P)\,sin\,\alpha + (X_c - X_P)\,cos\,\alpha \quad (13)$$

$$Y_{cb} = -b' + (-(X_c - Y_P) + (a' + X_{cb})\,cos\,\alpha) / sin\,\alpha \quad (14)$$

$$Z_{cb} = Z_c - Z_P \quad (15)$$

V. Simulations

Our approach was tested by simulation. The imposed operational task consists in following an ellipsoidal form (150cmx100cm) in a frame $(O_A, \vec{x}_A, \vec{y}_A)$ with the end-effector; the distance between successive step points is 1cm and the height grows with 1mm/step point (Fig.7), the heights are respectively 71.7cm and 150 cm for the first step point and the last one, knowing that the height of the platform is Z_P=40cm; hence the number of step points is 719.

The calculations of the mobile manipulator configurations are done using the following initial parameters: v=5cm/s, L=40cm, the maximum value of the directing angle for the front wheels is φ_{max}=0.35 radians, $Ang^0=[q_1^0\ q_2^0\ q_3^0\ q_4^0]^T=[0.5\ 1.5\ 0\ 0.14]^T$ radians. The admitted errors ε for all the generalised coordinates is fixed to 0.0009 radians, and the number of iterations is k_{max}=1000.

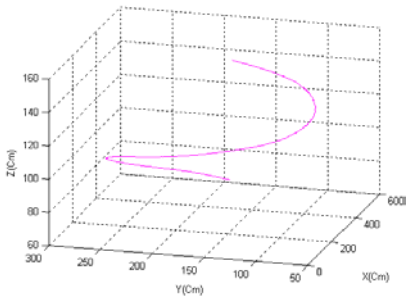

Fig.7.The imposed operational trajectory

The first phase in the proposed algorithm is the path planning with considering the distance *Dist*=20cm separating every couple of step points. In the following part, we show an example representing the fields of two step points.

A. Representing the fields

The different fields generated by the operational step points have a relationship with the height Z_c of the step point *C*.

Fig.8 shows an example representing the fields of two step points, the first one is from the 1st step point and the other is from the 550th one.

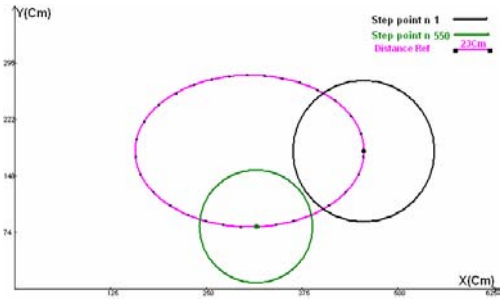

Fig.8. Representing the fields of two step points

We can notice that the step point number 1 has a radius of its field greater than the radius from the 550th step point, this is relative to the effect of the step point's height Z_c, because the first step point is higher than the 550th one.

B. Trajectory and motion planning for the mobile manipulator

In the trajectory planning, we have obtained different reference points P_k with modifying the method presented in paragraphII.C, we start by a first step point C_1 into the last one C_N, then we have done the opposite procedure that is starting by the last step point C_N until C_1, the aim of this strategy is to obtain a maximum number of reference points P_k, in this way we produce a regular variation of angles between two connecting segments τ_i, and we avoid some too abrupt changes of φ, and consequently, modifying this angle in regular manner. This approach will be called a « *round-trip planning* ».

The results are presented on Fig.9, which shows that the planned path of the car comes nearer to the operational trajectory, proportionally to the augmentation of a required height. The resulting trajectory of the mobile robot is different from the planned path, because nonholonomic constraints are included in equations (10-12), we also notice that the real path is moved forward to the planned trajectory in the lower part of the ellipsoidal operational trajectory; from this, the distance D_{BE} is greater than R_{max}, and the end-effector of the carried arm could not reach the required step points, in order to resolve this problem, we have decreased the radius of all generated fields, this insures a correct following of the operational trajectory by the manipulator.

The initial car's starting position has been done with respecting the distance R_{max} between O_{B0} and the first step point C_1 , in addition, we have fixed α so that it have the same value of the first segment orientation τ_1 (Fig.9). In general, during the motion, both systems of manipulation and mobility react in the same way; when the step point can be reached by the two systems (manipulator and platform) their motions are done simultaneously, otherwise, each the platform is solicited alone.

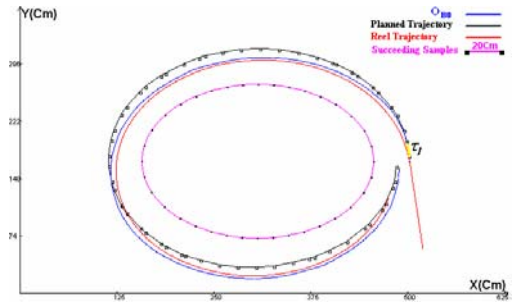

Fig.9. trajectory and motion planning of a mobile platform

The orientation α of the platform and the steering angle *φ* during the motion are plotted in Fig.10, according to the time *t*(s). We can notice that the steering angle in presents a considerable number of variations, it is a consequence of the round-trip path planning which generates a reduced length segments.

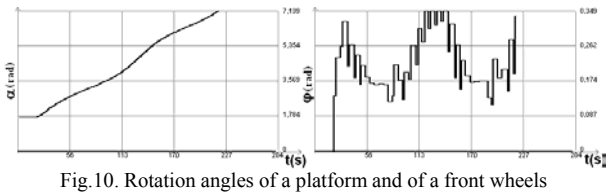

Fig.10. Rotation angles of a platform and of a front wheels

The second observation we can note on figure10 is relative to the straight vertical line (on the first 20.2 seconds), it's the effect of the straight line τ_1 followed by a platform (Fig.9) where the rotation angle α is constant. From a given motion platform planning , we can know the real position of the origin O_{B0} in R_A, then; the inverse kinematic iterative method can be used to calculate the inverse kinematic model of the manipulator.

The results of the inverse kinematic model are presented in Fig.11 where the generalised coordinates of the arm are plotted according to the time *t*(s).

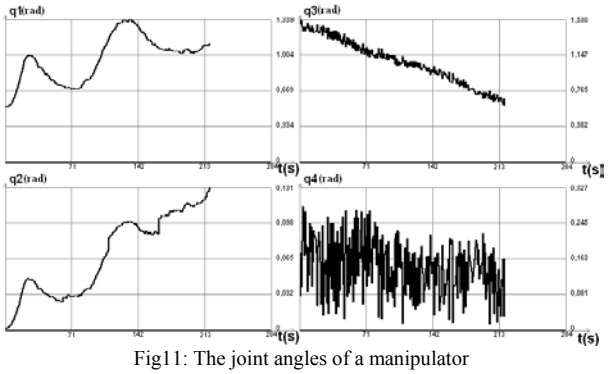

Fig11: The joint angles of a manipulator

The procedure to validate the kinematic model is shown in Fig.12, the vector A_D corresponds to the operational coordinates after the validation step; the result of the forward kinematic model is given in Fig.13.

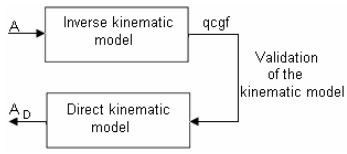

Fig.12. Validation of the inverse kinematic model

The optimal error in the operational space between desired results and the calculated ones from all step points are: $e_x=\max\|X_c-X_{calc}\|=0{,}033$cm, $e_y=\max\|Y_c-Y_{calculated}\|=0.064$ cm, $e_z=\max\|Z_c-Z_{calculated}\|=0.0018$cm, we can notice that all errors are lower than 1 mm.

The inverse kinematic model of the manipulator is calculated for every movement of the platform when the current step point is susceptible to be reached. For each configuration i+1, the ith configuration is its estimated one.

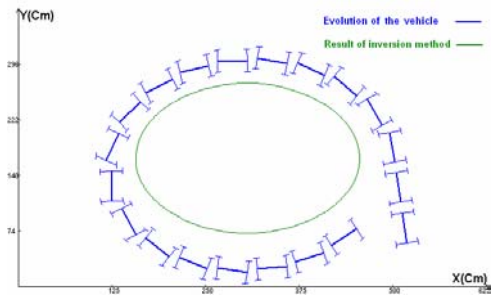

Fig13. Results of the inverse kinematic model and a following trajectory from a platform in $(O_A, \vec{x}_A, \vec{y}_A)$

VI. Conclusion

The main advantage of mobile manipulator systems is the enlarging of the workspace, because they can accomplish a manipulation task in significant surfaces, in the same time, their drawbacks is their redundancy included by the platform when the required task is the following of an operational trajectory by the end-effector. Indeed, for reaching an imposed step point in an operational space, it exists an infinity of configurations of the platform $[X_P\ Y_P\ \alpha]^T$. An other problem is due to nonholonomic constraints which limit the motion of the mobile system; the manipulator contributes also in redundancy with adding degrees of freedom.

In this paper we proposed a solution to a motion planning of a nonholonomic mobile robot carrying a redundant manipulator for following a trajectory which lies in the operational space.

The principle of the proposed methodology is to let the motion of the mobile system be prior with always considering the existence of the carried manipulator and to make it able to follow the operational trajectory.

The algorithm of motion planning is efficient for following a helical trajectory in a free space (not constrained by obstacles), however, it can be used in the case of an environment containing obstacles (for the mobile platform).

We have noticed that the evolution of the system is closely bounded to parameters that can influence the final result.

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