Abstract—This paper presents a comparative study between two different control strategies to determine the compensating currents for a four-wire shunt active filter. The two strategies are based on the time domain, the first is based on the instantaneous real and imaginary powers theory known as pq-theory, the second is based on the instantaneous currents (dq-currents). The two control strategies are applied to a four-wire shunt active filter working under sinusoidal and non-sinusoidal, balanced and unbalanced voltages. Simulation results and interpretations are presented to compare the performances of the two control strategies.

Index Terms—Four wires shunt active filter, pq-theory, dq-current, Harmonic compensation, PLL.

I. INTRODUCTION

Nowadays, many efforts are expended to develop power quality conditioners that can soften power quality problems (harmonics, Flicker, voltage sags...), caused by the proliferation of non-linear loads (especially power electronics equipment).

Active filters have come into view around 1970, after that, the concept has been successfully developed, tested and assisted by the power electronics technology, and put in practical uses [1][2]. In term of topology, three-phase filters can be divided into three-wire and four-wire active filters. The first one is well developed and put in industrial applications, however, in last years, researches are more oriented to the second one for its advantage in four-wire distribution systems.

This paper focuses on the four-wire three-phase shunt active filter for current compensation, using the conventional voltage three legs source inverter as illustrated in Fig.1, the neutral wire is connected directly at the midpoint of the DC bus.

An important part of this active filter is its control strategy. Many kinds of control strategies for shunt active filter have been treated so far and can be divided in two approaches the frequency domain and the time domain [3]. The second approach is interesting in term of calculation effort and identification quality.

In this paper, two control strategy based on the second approach are compared. The first is based on the instantaneous power proposed by Akagi in 1983 [4][5] for three-wire systems and expanded for four-wire systems [6]. However, the second control strategy is based on instantaneous currents. The objective of this study is to compare the performance of each one to compensate all perturbations on currents (harmonics, unbalance and neutral current) under non-sinusoidal and unbalanced voltages, and we will focus especially on calculation effort reduction.

II. CONTROL STRATEGY BASED ON THE PQ THEORY

It consists of compensating current identification by using instantaneous real and imaginary power theory proposed by Akagi in 1983 [4][5] for three-wire three-phase systems. It is an important tool to compensate harmonics and power factor improvement. This theory was expended for four-wire applications in [6][7]. Currently this control strategy is largely used in industrial compensators, not only in active filters but also in FACT-Systems [8], but need that the network voltages
measured at the common coupling point must have sinusoidal and balanced waveforms. However, it is important to signal that, this method cannot identify directly the harmonic components of the load current, but identifies the harmonic components which are not contribute to active power exchange between the source and the load. Thus if there is coincidence between these harmonics and those of network voltage, there will an exchange of active power and therefore they will not be identified as perturbation. Fig.2 illustrates the evaluation error on harmonic currents in the case of sinusoidal balanced voltages (right), and in the case of perturbed voltages (left) with a rate of harmonic distortion (THDv = 10%) and rate of unbalance (τr = 10%).

So to solve this problem, the control algorithm may be modified by introducing fundamental positive sequence identification in order to achieve correct compensation objectives in perturbed voltage situation.

The instantaneous homopolar, real and imaginary power are defined in matrix form by the following expression:

\[
\begin{bmatrix}
p_0(t) \\
p(t) \\
q(t)
\end{bmatrix} =
\begin{bmatrix}
e_0(t) & 0 & 0 \\
0 & e'_\alpha(t) & e'_\beta(t) \\
0 & -e'_\beta(t) & e'_\alpha(t)
\end{bmatrix}
\begin{bmatrix}
i_0(t) \\
i_\alpha(t) \\
i_\beta(t)
\end{bmatrix}
\]

(1)

Where \( i_0(t), i_\alpha(t) \) and \( i_\beta(t) \) are the instantaneous load currents \( i_{abc}(t) \) obtained using the following transformation:

\[
\begin{bmatrix}
i_0(t) \\
i_\alpha(t) \\
i_\beta(t)
\end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix}
1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\
1 & -1/2 & -1/2 \\
0 & \sqrt{3}/2 & -\sqrt{3}/2
\end{bmatrix}
\begin{bmatrix}
i_0(t) \\
i_\alpha(t) \\
i_\beta(t)
\end{bmatrix}
\]

(2)

\( e_0(t) \) is the homopolar component of the measured voltage, while \( e'_\alpha(t) \) and \( e'_\beta(t) \) are the instantaneous voltage components in \((\alpha, \beta)\) coordinate system, corresponding to the fundamental positive sequence of the measured voltage \( e_{abc}(t) \), these components are obtained using a PLL (Phase Locked Loop) system, more details are given later about this system.

If we consider that the fundamental positive sequence is correctly identified then, the instantaneous homopolar, real and imaginary powers are composed of continuous and alternative parts related respectively to fundamental and perturbations (harmonics, inter-harmonics, unbalance) of the measured voltage and current.

\[
\begin{align*}
p_0(t) &= p_0(t) + \bar{p}_0(t) \\
p(t) &= \bar{p}(t) + \bar{p}(t) \\
q(t) &= \bar{q}(t) + \bar{q}(t)
\end{align*}
\]

(3)

Where:

\( p_0(t) \) is the mean value of the instantaneous homopolar power, it corresponds to the energy per time unity which is transferred from the source to the load through homopolar components of voltage and current.

\( \bar{p}_0(t) \) is the alternative value of the instantaneous homopolar power, it means the energy per time unity that is exchanged between the source and the load through homopolar components of voltage and current. Note that homopolar power exists only in three phase systems with neutral wire.

\( \bar{p}(t) \) is the mean value of the instantaneous real power corresponding to the energy per time unity transferred between the source and the load through voltage and current harmonics which have the same frequency and the same sequence.

\( \bar{q}(t) \) is the alternative value of the instantaneous real power corresponding to the energy per time unity exchanged between the source and the load through voltage and current harmonics which have different frequencies and/or sequences.

The imaginary power \( q(t) \) represents an energy that is exchanged between the phases of the system. This means that \( q(t) \) does not contribute to the energy transfer between the source and the load, its alternative value \( \bar{q}(t) \) corresponds to the energy per time unity exchanged through voltage and current harmonics which have the same frequency and the same sequence, and its alternative part \( \hat{q}(t) \) corresponds to the energy per time unity exchanged through voltage and current harmonics which have different frequencies and/or sequences.

The compensating currents in \((\alpha, \beta)\) coordinate are calculated using the \( \bar{p}(t), \bar{p}_0(t) \) and \( q(t) \) as shown in (4).

\[
\begin{bmatrix}
i^*_{\alpha}(t) \\
i^*_{\beta}(t)
\end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix}
e'_\alpha(t) & -e'_\beta(t) \\
e'_\beta(t) & e'_\alpha(t)
\end{bmatrix}
\begin{bmatrix}
\bar{p}(t) - \bar{p}_0(t) \\
q(t)
\end{bmatrix}
\]

(4)

where \( \Delta = e'^2_\alpha(t) + e'^2_\beta(t) \). Otherwise, the homopolar reference compensation current is \( i^*_{\alpha} = i_0 \).

Finally, the reference compensating currents in \( abc \) coordinates are given by (5).

\[
\begin{bmatrix}
i^*_{\alpha}(t) \\
i^*_{\beta}(t) \\
i^*_{\gamma}(t)
\end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix}
1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \\
1 & -1/2 & -1/2 \\
0 & \sqrt{3}/2 & -\sqrt{3}/2
\end{bmatrix}
\begin{bmatrix}
i^*_{\alpha}(t) \\
i^*_{\beta}(t) \\
i^*_{\gamma}(t)
\end{bmatrix}
\]

(5)

The block diagram of this control strategy is shown in Fig.3. The \( V_{DC} \) regulation is included in this block, in order to generate an auxiliary control signal \( P_{loss} \), this one is handled in the active filter controller as an active power which forces the voltage source inverter to draw or to inject from/in the network in order to keep \( V_{DC} \) around of its reference \( V_{DC}^* \).

III. VOLTAGE FUNDAMENTAL POSITIVE SEQUENCE IDENTIFICATION

To identify fundamental positive sequence of the network voltage, we have used an algorithm based on the real and imaginary power theory [7]. An important part of this algorithm is the PLL system which determines continuously the fundamental angle \( \hat{\omega}t \) (see Fig.5), the PLL precision determines the quality of fundamental positive identification. The frequency at PLL output is used to generate auxiliary
Fig. 3. Block diagram of the pq control strategy for four-wire Shunt active filter.

Fig. 4. Block diagram of the pq control strategy for fundamental positive sequence identification.

Fig. 5. PLL system for pq control strategy.

Fig. 6. Fundamental positive sequence and its frequency extracted since distorted and unbalanced voltages.

PLL System

The PLL system used to extract the fundamental pulsation is shown in Fig. 5, it is based on the pq-theory [9]. The instantaneous real power in the PI-controller input is:

$$p_{PI}(t) = \frac{1}{2}(e_\alpha(t)i'_\alpha(t) + e_\beta(t)i'_\beta(t))$$

These currents are used with the measured voltage to determine instantaneous real power $p(t)$ and instantaneous imaginary power $q(t)$. Thus these powers contains all information about measured voltage, and the mean values $p(t)$ and $q(t)$ correspond to the fundamental positive sequence, so we use them to identify in the ($\alpha, \beta$) frames $e_\alpha(t)$, $e_\beta(t)$ corresponding to the fundamental positive sequence.

$$\begin{bmatrix} e'_\alpha(t) \\ e'_\beta(t) \end{bmatrix} = \frac{1}{i''_\alpha(t)^2 + i''_\beta(t)^2} \begin{bmatrix} i''_\alpha(t) & i''_\beta(t) \\ i''_\beta(t) & -i''_\alpha(t) \end{bmatrix} \begin{bmatrix} p(t) \\ q(t) \end{bmatrix}$$

(7)

This means that the stable point of operation is found only $\dot{\omega}$ correspond to the fundamental frequency of the system and the auxiliary signals $i'_\alpha(t)$ and $i'_\beta(t)$ become orthogonal to the measured network voltages $e_\alpha(t)$ and $e_\beta(t)$ respectively.

Fig. 4 shows the block diagram of this identification method.

IV. CONTROL STRATEGY BASED ON THE INSTANTANEOUS CURRENTS

This method, also called synchronous method [10], consists to transform currents in a turn coordinate system by using Park transformation [11], with the same effect as the instantaneous power method. In fact, if the rotating angle correspond to the fundamental frequency, then currents $i_{dq}$-axis are composed by a DC component (related with fundamentals) and AC components (with harmonics), which can be separated easily.
The active power is obtained in the neutral wire, \( \bar{i} \), where
\[
\begin{bmatrix}
i_d(t) \\
i_q(t)
\end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \omega t & \sin \omega t \\ 0 & \cos \omega t & -\sin \omega t \end{bmatrix} \begin{bmatrix} i_0(t) \\
i_\alpha(t) \\
i_\beta(t)
\end{bmatrix}
\]

Where \( i_0 \) correspond to the homopolar power flowing in neutral wire, \( i_d \) correspond to the reactive power and \( i_q \) to the active power.

\[
i_0(t) = \bar{i}_0(t) + \tilde{i}_0(t)
\]
\[
i_d(t) = \bar{i}_d(t) + \tilde{i}_d(t)
\]
\[
i_q(t) = \bar{i}_q(t) + \tilde{i}_q(t)
\]

In general, the rotation angle used in the Park transformation is given by a PLL since network voltages, in this paper in order to reduce the number of measures, we have proposed to extract the rotation angle on load currents. The Fig.7 illustrates the identification block diagram of this control strategy.

The reference compensating currents are calculated by the non desired parts of the currents \( i_0 \), \( i_d \), and \( i_q \) to achieve compensation objectives, if we want to compensate exclusively harmonics, then \( i_0 \), \( i_d \), and \( i_q \) are used to calculate reference compensating. For a total compensation, we chose \( i_\alpha, i_\beta \), and \( i_q \) and the \( abc \)-compensating currents are given using the inverse Park transformation (12).

\[
\begin{bmatrix}
i_\alpha(t) \\
i_\beta(t) \\
i_q(t)
\end{bmatrix} = \sqrt{\frac{2}{3}} A \begin{bmatrix} i_0(t) \\
i_\alpha(t) \\
i_q(t)
\end{bmatrix}
\]

Where
\[
A = \begin{bmatrix} 1/\sqrt{2} & \cos \omega t & -\sin \omega t \\ 1/\sqrt{2} & \cos(\omega t - 2\pi/3) & \sin(\omega t - 2\pi/3) \\ 1/\sqrt{2} & \cos(\omega t - 4\pi/3) & \sin(\omega t - 4\pi/3) \end{bmatrix}
\]

### PLL system

The PLL circuit used in the \( dq \)-current control strategy is shown on Fig.8, the operation principle is the same as the one used in the \( pq \) control strategy, but the input parameters are load currents transformed in \((\alpha, \beta)\) frames. The PLL tracks continuously the pulsation of the fundamental positive sequence of the load currents.

An important remark must be signaled here, that in general load current are very perturbed, therefore, to reduce oscillation in the identified pulsation the cut frequency of the PLL system must be chosen as low as possible. In Fig.9, we illustrate the identification of the fundamental angle \( \theta \) since load currents.

The main advantage in this control strategy is that the compensating currents identification is correctly achieved independently of the network voltage situation. The active filter controller needs information only about load currents. It is a robust identification method, Fig.10 shows that it is insensitive to voltage distortion or unbalance.

### V. Simulation Results

The circuit configuration in Fig.1 is implemented in MATLAB/SimPowerSystems environment ; we have considered unbalanced and non sinusoidal voltage as it can be seen in Fig.12, and a non linear load absorbing a very perturbed current from the network. The main parameters of this circuit are recapitulated in Table I.

The current at the VSI is controlled using PI-Controller with a PWM modulator [12].

Fig.11 shows the abc-phases current forms and the current in the neutral wire. In Fig.13 the performance of the instantaneous power theory is shown.

Fig. 7. Block diagram of the \( dq \) Control strategy.

Fig. 8. PLL System for \( dq \)-currents control strategy.

Fig. 9. Fundamental angle extracted from perturbed load currents.

Fig. 10. Harmonics estimation error in the \( dq \) control strategy, perturbed voltage (left), sinusoidal and balanced voltage (right).
Table I

Main parameters of the simulated circuit.

<table>
<thead>
<tr>
<th>Network</th>
<th>Unbalance rate: ( \tau = 10% )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total harmonic distortion: ( T H D_{\varphi} = 3.4% )</td>
<td></td>
</tr>
</tbody>
</table>

| Load | Six-pulses diode bridge rectifier. |
|      | Four-pulses thyristor bridge rectifier between a-phase and the neutral. |
|      | Four-pulse diode bridge rectifier between b-phase and the neutral. |

| Active filter | Source: \( V_{DC}/2 = 400\text{V} \) |
|              | Passive filter: \( L = 5\text{mH} \) |
|              | PWM frequency: \( 10\text{kHz} \) |

Simulation results for the instantaneous currents control strategy developed here to four-wire shunt active filters control are shown in Fig. 14, we can remark that no significative difference between its performances with the instantaneous power control strategy control strategy. The source current is almost sinusoidal and balanced, the current in the neutral wire is compensated correctly with this method and the power oscillations are much reduced.

Harmonic components in the three-phase source currents are almost eliminated as it can be seen in Table III. Table II shows the total harmonic distortion for the \( abc \)-phase load and source current, we can remark that the THDs of the source currents are reduced considerably, note that here take

![Fig. 11. Load currents and neutral wire current.](image)

![Fig. 12. Instantaneous voltage corresponding to fundamental positive sequence extracted by the PLL system.](image)

![Fig. 13. Performance of the instantaneous power control strategy: phase compensating currents, neutral compensating current, source current, neutral source current and instantaneous three-phase active power.](image)

Table II

Total harmonic distortion of the three-phases for load and source currents.

<table>
<thead>
<tr>
<th>Phase</th>
<th>Load current THD (%)</th>
<th>Source current THD (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>26.94</td>
<td>02.63</td>
</tr>
<tr>
<td>( b )</td>
<td>17.99</td>
<td>02.57</td>
</tr>
<tr>
<td>( c )</td>
<td>29.58</td>
<td>02.59</td>
</tr>
</tbody>
</table>

Table III

Harmonics components for the three-phase load and source currents.

<table>
<thead>
<tr>
<th>Harm</th>
<th>Load ( \text{ph} \ a )</th>
<th>Source ( \text{ph} \ a )</th>
<th>Load ( \text{ph} \ b )</th>
<th>Source ( \text{ph} \ b )</th>
<th>Load ( \text{ph} \ c )</th>
<th>Source ( \text{ph} \ c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26.45</td>
<td>25.40</td>
<td>29.39</td>
<td>25.19</td>
<td>20.72</td>
<td>25.23</td>
</tr>
<tr>
<td>3</td>
<td>03.66</td>
<td>01.08</td>
<td>01.23</td>
<td>01.07</td>
<td>01.57</td>
<td>01.84</td>
</tr>
<tr>
<td>5</td>
<td>04.22</td>
<td>00.08</td>
<td>03.54</td>
<td>01.97</td>
<td>04.73</td>
<td>00.67</td>
</tr>
<tr>
<td>7</td>
<td>02.79</td>
<td>06.63</td>
<td>02.61</td>
<td>01.78</td>
<td>02.18</td>
<td>00.74</td>
</tr>
<tr>
<td>9</td>
<td>01.91</td>
<td>05.55</td>
<td>01.46</td>
<td>03.83</td>
<td>00.73</td>
<td>00.31</td>
</tr>
<tr>
<td>11</td>
<td>01.27</td>
<td>07.00</td>
<td>00.43</td>
<td>00.30</td>
<td>01.80</td>
<td>00.83</td>
</tr>
<tr>
<td>13</td>
<td>00.86</td>
<td>05.05</td>
<td>01.18</td>
<td>00.70</td>
<td>01.04</td>
<td>00.72</td>
</tr>
</tbody>
</table>
in account high frequency harmonics generated by the PWM current control. The harmonics in the instantaneous active power are eliminated as shown in Table IV.

VI. CONCLUSION

A comparative study between two different control strategies for four-wire shunt active filters control is presented in this paper, considering disturbed network voltages.

The original pq control strategy compensates the harmonic components that do not contribute to active power exchange and cannot identify correctly perturbation in presence of perturbation on voltage. Therefore, to compensate all harmonics, this control has been modified with introducing the fundamental positive sequence extraction based also on the same theory.

The control strategy based on the instantaneous currents introduced in this paper identifies correctly harmonics and reactive current independently of the voltage. The simulation results shown that it can be applied successfully to achieve compensation objective in four-wire distribution systems. Otherwise, comparatively with the modified pq control, we can say that there is no significant difference in terms of results but it presents more advantages in term of calculation efforts. With this method, we have reduced considerably the calculation effort by introducing a simple and robust PLL system which can extract fundamental frequency on the load currents, so, we have eliminated the need of voltage measurement to identify load harmonics, and then the control algorithm is very simplified.

TABLE IV

<table>
<thead>
<tr>
<th>k</th>
<th>Load $P_k/P_0$ (%)</th>
<th>Source $P_k/P_0$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>11040</td>
<td>11140</td>
</tr>
<tr>
<td>2</td>
<td>1828</td>
<td>16.55</td>
</tr>
<tr>
<td>4</td>
<td>643.9</td>
<td>85.83</td>
</tr>
<tr>
<td>6</td>
<td>757</td>
<td>85.64</td>
</tr>
<tr>
<td>8</td>
<td>98.28</td>
<td>87.981</td>
</tr>
<tr>
<td>10</td>
<td>308</td>
<td>2.78</td>
</tr>
<tr>
<td>12</td>
<td>572.2</td>
<td>0.47</td>
</tr>
</tbody>
</table>

REFERENCES